fixturing technology, active fixture,
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CONTROL OF A WORKPIECE HOLDER WITH PIEZO-ELECTRIC-MECHANICAL ACTUATION

The main focus of research in part-fixture mechanics has been in the static deformation and part constraints. Little attention has been paid to the dynamic behaviour of these systems. However, the dynamics of the system largely determines the obtained precision during the machining process. The part is modelled as a lumped mass-spring model and the clampers and locators of the fixture are modelled as spring-dampers. The fixture frame is considered to be much stiffer than the locators, such that it provides zero displacement boundary conditions to the locators. In this study a piezoelectric actuator is utilized to provide adaptive clamping forces. The analysis has shown that position feedback can be used to minimize unnecessary displacement of the workpiece. Additionally, a lag filter can improve the steady state response.

1. INTRODUCTION

Fixtures, are used to fixate, position and support workpieces, and form a crucial tool in manufacturing. Their performance influences the manufacturing (and assembly) process of a product. A vast amount of research has been made on fixturing technology (see e.g.: [11,15]). The main focus of research in part-fixture mechanics, however, has been in the static deformation and part constraints [11,15]. Little attention has been paid to the dynamic behaviour of these systems. Nevertheless, the dynamics of the system largely determines the obtained precision during the machining processes.

In order to provide restraint and support, the fixture needs to be stiff. Especially flexible parts might need additional support, such that the part undergoes minimal deformation due to the machining process. The deflection of the part-machine system determines the obtained precision of the manufacturing process. Controlled actuators can be used to provide the support and constraint in an effective manner. According to the best

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knowledge of the authors, little attempts [11, Chapter 6] have been made for the application of control engineering in the design of fixtures.

Mittal *et al.* [9] established a model whereby spring-dashpot elements are used to describe the (contact) stiffness of the clampers and locators, based on the papers by Shawki and Abdel-Aal [16,17]. The approach avoids “computationally expensive” contact mechanics in the model. It has been widely adopted in the manufacturing research community, see e.g. [15], and will be used in this paper.

Fixtures are actuated by several methods. On of these methods is the application of a piezoelectric actuator (PEA) [5,8,14]. PEAs can be used for the control of vibration in a system and additionally to introduce more system damping [10,13,14]. Moreover PEAs are sometimes adopted in positioning systems where a high level of accuracy is required [2,13]. In this work a PEA is utilized for the actuation of the fixture.

The paper aims to present an application of a PEA for feedback control of an active part-fixture system, in order to make the fixture system stiffer compared to conventional design. The paper can be considered to be the logical sequel of [4], where a model of a hydraulic actuator without valve and a part, modelled as a rigid body with a mass, is presented. Also, of [3], where a methodology to obtain a model of an active fixture coupled to a reduced model of a flexible part is demonstrated, using a more detailed modelling of the hydraulic actuator. In the current study, the workpiece is considered to be a rigid body with a mass. A model of a motion amplifier as a fixture representation and a representation of the dynamics of a PEA are established. Subsequently, in Section 2.6 several control strategies are further considered. An example is given in Section 3. The main findings of this research are presented in Section 4.

2. METHODOLOGY

In this section, the PEA, the part and the fixture, represented as a lever mechanism acting as a motion amplifier, are modelled.

2.1. MODEL OF AN ACTUATED PART-FIXTURE SYSTEM

In Fig. 1 one can see a simplified model of an actuated part-fixture system. The PEA is connected to a motion amplifier. For the *static case* this means that the forces exerted by the PEA will be experienced by the part as only a fraction r_a / r_c of them. The actuator has a displacement x_p, obviously due to the coupling to the lever, as a result the rotational displacement is $\theta = x_p / r_a$. In order to make the model more realistic the lever has rotational inertia I, at the same time the lever is considered to be rigid. In this paper the part is considered to be a lumped mass, i.e.: a rigid part that has a mass M. It is connected to the fixture by means of a spring-dashpot element, the locator is assumed to have the same stiffness and damping values as the connection between lever and part, for simplicity and without any loss of generality. Furthermore a machine force F_m is applied to the part.

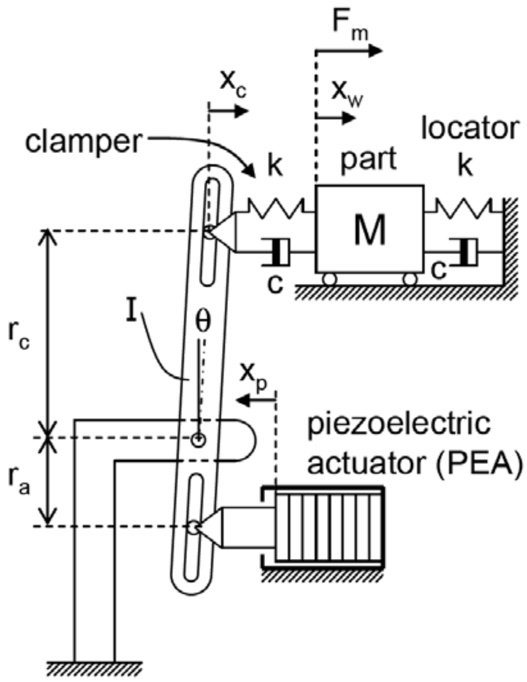

Fig. 1. Model of the actuated part-fixture system

2.2. ACTUATOR

The coupled electro-mechanical behaviour of a stacked PEA is usually modelled using the standards established by the IEEE [1], this approach can be found in many publications and standard textbooks such as [10,13]. In Fig. 2 a model of a stacked PEA is given; it explains the modelling as presented in [2] (and by Goldfarb and Celanovic [7]).

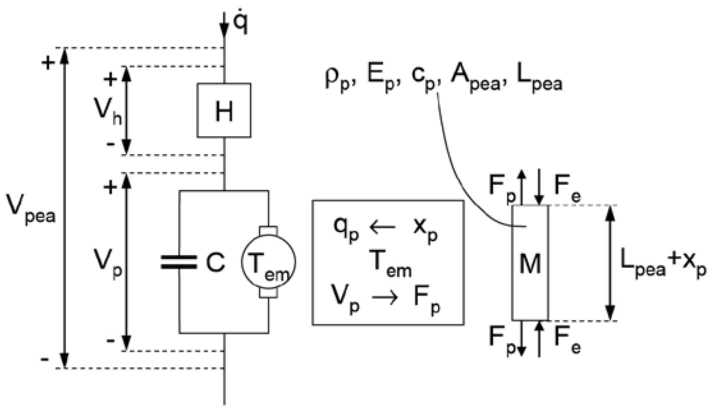

Fig. 2. Electro-mechanical model of a stacked piezoelectric actuator (after [2])

Adriaens *et al.* [2] split the behaviour of the PEA in two parts that are modelled in series. V_{pea} is the overall voltage that is applied over the PEA. A part of this voltage, V_h, drops due to the hysteresis effects in the electro-dynamic behaviour of the piezoelectric materials, which are usually manmade ceramics like PZT (Lead Zirconate Titanate). The other part of the total applied voltage drops across the piezo-electro-mechanical transducer. This transducer relates the charge q_p and voltage over the PEA V_p to respectively the actuator displacement x_p and the force F_p exerted by the actuator by a transformer ratio T_{em}. This is called the piezo-effect and can be seen in the middle and righthand side of Fig. 2. Before going into the mathematical details of the actuator model it needs to be mentioned that if charge feedback is applied on the PEA, the hysteresis effects are effectively cancelled [2,10]. It is therefore assumed in this study that charge feedback is applied to control the PEA and that the hysteresis effects do not need to be modelled and hence the voltage across the piezo-electro-mechanical transducer equals the voltage that has been applied over the PEA: $V_p = V_{pea}$.

The electro-mechanical coupling equations read [1,10,13]:

$$\begin{cases} \{\varepsilon_i\} = [S_{ij}^E]\{\sigma_j\} + [d_{im}]\{E_m\} \\ \{D_m\} = [d_{mi}]\{\sigma_i\} + [\xi_{ik}^{\sigma}]\{E_k\}. \end{cases} \tag{1}$$

where $\{\varepsilon_i\}$ is the strain vector, $[S_{ij}^E]$is the compliance matrix, $\{\sigma_j\}$ the stress vector, $[d_{im}]$ is the matrix of piezoelectric strain coefficients, $\{E_k\}$ ($\{E_m\}$) is the vector of the applied electric field, $\{D_m\}$ is the vector of the electric displacement and $[\xi_{ik}^{\sigma}]$ is the permittivity matrix, also $[d_{im}] = [d_{mi}]^T$. Using (1) it is obvious that the charge q_p and voltage over the PEA V_p are related to respectively the actuator displacement x_p and the force F_p exerted by the actuator by the transformer ratio T_{em}, which is expressed as:

$$T_{em} = d_{33} k_p, \tag{2}$$

where d_{33} is the piezoelectric strain coefficient in use when the piezoelectric elements are in this configuration, and k_p is the stiffness of the PEA which is known to be $k_p = E_p A_{pea} / L_{pea}$ The transformer coefficient T_{em} can sometimes also be found in the online catalogues of PEA manufacturers, e.g. [12]. Goldfarb and Celanovic [7] followed by Adriaens *et al.*[2] realized that the action of a stacked PEA that is of interest to the designer is only uniaxial, namely the third axis, and substituted the *static* constitutive relationships $[S_{ij}^E]\{\sigma_j\}$ by the *dynamic* equation of motion in this direction. Furthermore using the transformer coefficient, the equation of motion of the PEA becomes as follows [2, 7]:

$$m_p \ddot{x}_p + c_p \dot{x}_p + \left(kp + \frac{T_{em}^2}{C} \right) x_p = F_e + \frac{T_{em} C_e}{B_2 C} V_{pea}, \tag{3}$$

where m_p is the effective mass of the PEA, c_p the damping in the piezoelectric ceramic material, C is the capacitance of the PEA, C_e and B_2 are respectively the capacitance and an amplification value in the charge feedback scheme. F_e is the external force, exerted on the PEA. This is a connection force that will be described in Section 2.5.

2.3. LEVER MECHANISM

The PEA is coupled to the lever, hence the actuator displacement x_p is directly coupled to the lever rotation θ. The top (clamper) displacement x_c is also related to the angular displacement θ. From Fig. 1 these relationships are easily established as follows

$$x_p = r_a \tan\theta,\ x_c = r_c \tan\theta. \tag{4}$$

Since $\dot{r}_a = \dot{r}_c = 0$ and for small angular displacements θ, these relationships can be linearized to

$$x_p = r_a\theta,\ x_c = r_c\theta,\ \dot{x}_p = r_a\dot{\theta},\ \dot{x}_c = r_c\dot{\theta},\ \ddot{x}_p = r_a\ddot{\theta}. \tag{5}$$

2.4. PART MODELLING

The part is modelled as a single mass connected by spring and dashpot to the actuated clamper and to the locator. Setting up the equation of motion for the part becomes therefore quite straightforward and is done as follows. The mass of the part M times its acceleration $\ddot{x}_w$ equals the machining force F_m and in case when the part moves to the right in Fig. 1, i.e. the positive direction of x_w, the restoring force exerted by the locator $-c\dot{x}_w - kx_w$ and the force applied by the clamper. Note that this is also the connection force between part and fixture $F_c = -c(\dot{x}_w - r_c\dot{\theta}) - k(x_w - r_c\theta)$. Substituting $\dot{x}_c$ and x_c with the relationships from Eq. (5) yields the following equation of motion:

$$M\ddot{x}_w = -c\dot{x}_w - c(\dot{x}_w - r_c\dot{\theta}) - kx_w - k(x_w - r_c\theta) + F_m. \tag{6}$$

2.5. ACTUATED FIXTURE

The moment equation of the lever mechanism about its pivot, which is similar to the positioning system as presented in [2], reads:

$$F_e = \frac{I}{r_a}\ddot{\theta} + F_c, \tag{7}$$

where F_e is the external force needed in (3) and the connection force F_c is defined in Section 2.4. Substituting (7) in (3) gives

$$\left(m_p r_a + \frac{I}{r_a}\right)\ddot{\theta} + \left(c_p r_a + \frac{r_c^2}{r_a}c\right)\dot{\theta} + \left(k_p r_a + \frac{T_{em}^2 r_a}{C} + \frac{r_c^2}{r_a}k\right)\theta = \frac{T_{em}C_e}{B_2 C}V_{pea} + \frac{r_c}{r_a}c\dot{x}_w + \frac{r_c}{r_a}kx_w. \tag{8}$$

2.6. CONTROL STRATEGIES

An investigation has been made into the use of position feedback: firstly, the actuator displacement (ADFB), secondly, the part displacement (PDFB), and, thirdly, force feedback (FFB) have been examined. The aim of this paper is to investigate classical control strategies in the form of series compensation. PEAs characteristically possess a poorly damped behaviour [2]. This means that at the resonance frequency the overshoot for a sinusoidal input is larger than +3 dB. This undesired response at the resonance frequency can be removed by using a compensator that acts as a low-pass filter. In Fig. 3 the closed-loop control system can be seen. The reference signal r is compared with one of the outputs of the system (x_p, x_w or F_{act}). The actuation force F_{act} is one of the standard measured outputs of a PEA [12] and is defined as $F_c r_c / r_a$. The error signal ε feeds into the controller $C(s)$ and the controller signal feeds into the 'plant', formed by the PEA, the motion amplifier and the part. The machining force Fm is a direct input in the 'plant'. The actuator and workpiece positions x_p and x_w and the actuator force F_{act} are the output of the system.

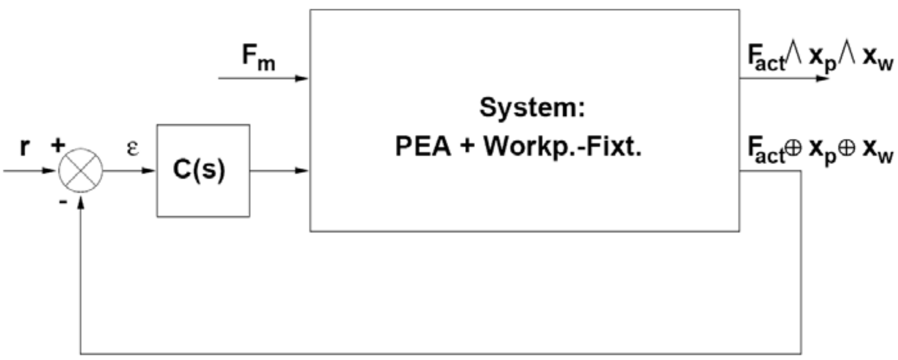

Fig. 3. Block diagram of control system; ∧ = "and", ⊕ = "or"

The closed-loop systems do not need to be compensated by a proportional-integral controller for steady state error. For this reason only proportional control [6] (P-control) and lag filter compensation (LaF) [6] are considered in this study. In case of P-control the transfer function *C(s)* of the controller is defined as follows [6]

$$C(s) = K_p. \tag{9}$$

A lag filter has the following transfer function

$$C(s) = K_p \frac{s + \omega_{LF}}{\alpha_{LF} s + \omega_{LF}}, \tag{10}$$

where $\alpha_{LF} > 1$. The proportional gain K_p, the settings for the corner frequencies $\omega_{LF} / \alpha_{LF}$, and ω_{LF} need to be tuned appropriately. These settings determine respectively the corner frequency for the maximum controller response and the minimum response from the compensator. The design rules for application of a LaF can be found in standard textbooks such as [6]. In order to increase the working of the controller, in case of actuator or part displacement feedback control a double LaF has been used as compensator.

3. RESULTS AND DISCUSSION

A representative and realistic system is demonstrated in this section. Firstly, the system's response to sinusoidal input, better known as the frequency response is investigated. Secondly, the systems transient behaviour is considered.

3.1. EXAMPLE

A numerical example has been made base on the model described above. The PEA used in the calculation is the P-239.90 from PI [12]. The damping cp in the PEA is thought to be proportional with its stiffness k_p and is taken from [2]. The properties of the system used in the example can be found in Table 1. It is assumed that in the charge feedback circuit the capacitance C_e equals the capacitance of the PEA. A second assumption is that the amplifier gain B_2 is equal to 1. A frequency response plot shows the amplitude of the response of a system output to a harmonic excitation with a fixed frequency of an input to the system. This response is measured or computed for a whole range of input frequencies, that allow to construct a frequency response diagram for this range [6]. The frequency response can have a delay that can be expressed as a certain number of degrees in phase change: input $\sin(\omega t)$, output: $\sin(\omega t+\varphi)$, where φ is the phase change. The system becomes unstable when the system has a response of 0 dB or higher and a phase change of π rad. In order to give an estimate of the stability of the system near a resonance frequency, two margins are defined that show how far the system is from this instability point. The gain margin is defined by how much the gain is lower than 0 dB when the phase change is π rad. The phase margin shows how far the phase change is away from π radians, when the gain is 0 dB.

Table 1. Properties of PEA and part-fixture system

Property	Symbol	Unit
Stiffness clamper/locator	k	0.7 MN/m
Damping coefficient clamper/locator	c	146.66 Ns/m
Mass part	M	0.168 kg
Radius lever	r_a	0.02 m
Radius lever	r_b	0.12 m
Inertia lever	I	0.0017 kgm^2
Transformer ratio	T_{em}	17.5 C/m
Stiffness PEA	k_p	35 MN/m
Effective mass PEA	m_p	0.2216 kg
Damping PEA	c_p	87.5 Ns/m
Capacitance PEA	C	2100 nC

3.2. FREQUENCY RESPONSE

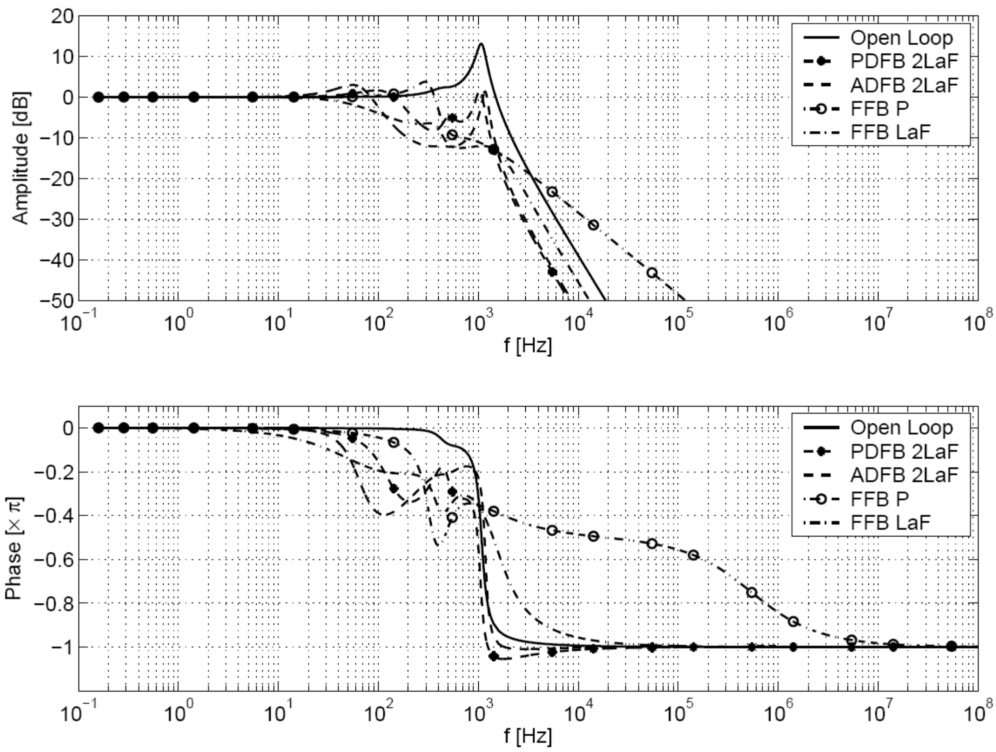

Fig. 4. Bode diagram for response of the actuator displacement x_p to input reference r, where r is the input reference voltage V_{pea}

It can be seen in Fig. 4 that the PEA behaves quite undamped in the open loop situation. However, its phase change is less than π rad, hence the system is stable. Actuator and part displacement feedback have a second resonance peaks between 1000 and 1200 Hz. In this case the phase margin is small, but the system does not become unstable. Using FFB creates a system that behaves in very stable manner. Force feedback control gives a smaller bandwidth compared to displacement feedback.

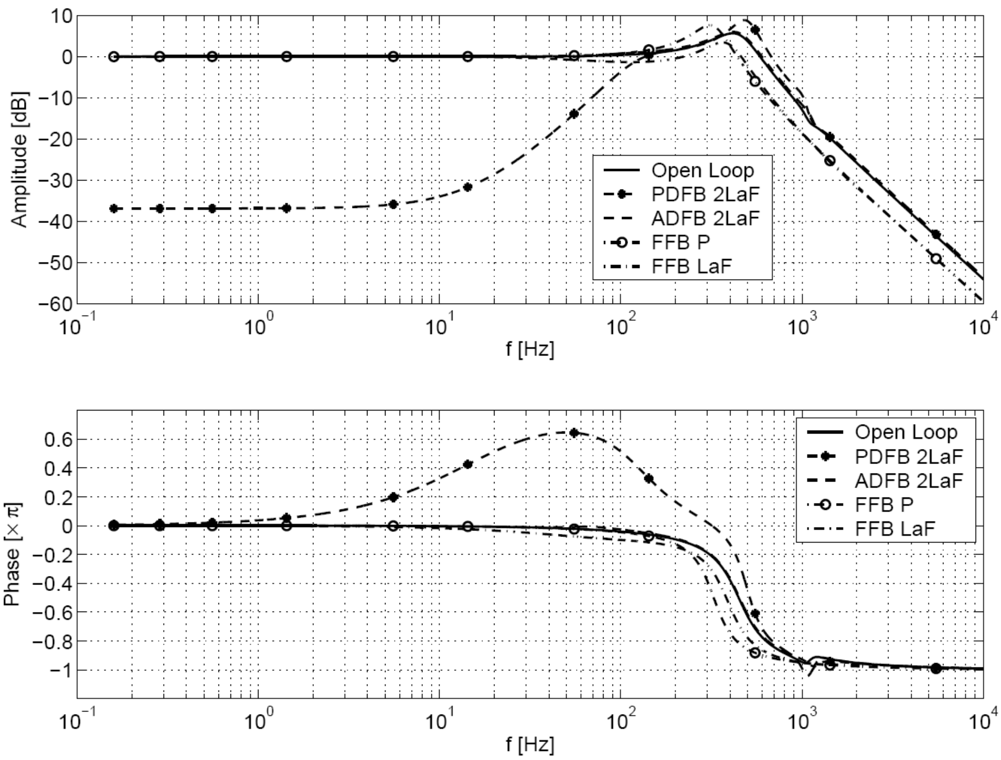

Fig. 5. Bode diagram for response of the part displacement x_w to machining force F_m

In Fig. 5 the response of the part displacement to the machine force is shown. In general the system behaves in a reasonably stable manner. One can observe a 30 dB gain margin in case of FFB and 25 dB for the rest of the analyzed control strategies. FFB control yields a phase margin of 0.1π rad and the other control strategies provide an even larger phase margin. FFB using proportional control gives 3.4 dB peak response, this is the smallest obtainable peak response. It can also be observed that for low frequencies the part displacement feedback has a low response for the machine force input, which is as expected.

3.3. TRANSIENT MACHINING SIMULATION

The system's transient behaviour is analyzed in this section. The part is clamped first, although this is not considered in detail in this study, and subsequently the clamped part-fixture system undergoes a series of machining operations, represented by the machining profile displayed in Fig. 6. Since the system frequencies are quite high the system's transient behaviour can be demonstrated in quite a short time span, which in the real world would be unrealistic. The steps in the machining force shown in Fig. 6 form an input that is composed by all frequencies. Therefore all eigenfrequencies present in the system will be excited which makes the machining force profile shown in Fig. 6 a good and “cheap” alternative representation for a real machining process like cutting, milling or grinding.

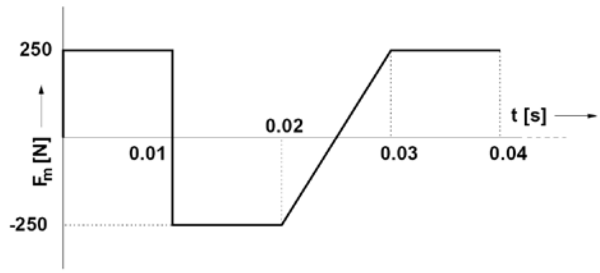

Fig. 6. Machine force profile

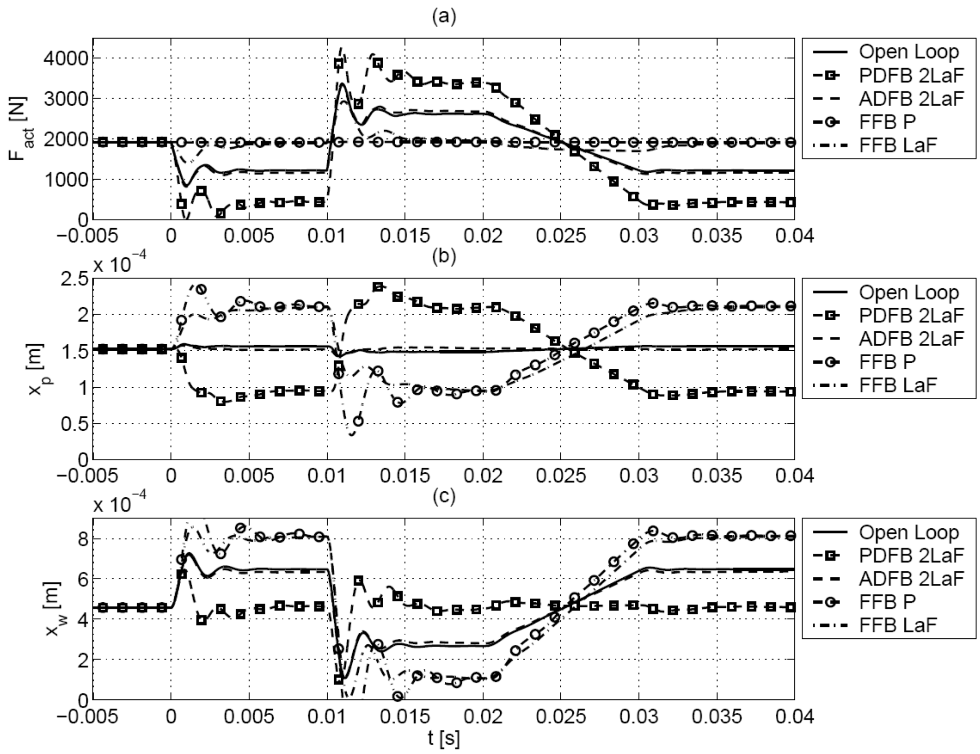

Fig. 7. Comparison for displacement under the same clamping force; machining time 0.04 s, clamping force at t = 0 s, = 320 N; with (a) dynamic response F_{act} to F_m, (b) dynamic response x_p to F_m, (c) dynamic response x_w to F_m

Several observations can be made. Firstly, the open loop control gives almost the same results as actuator displacement feedback control. Secondly, by part displacement feedback control, the part will has the smallest displacement. Also, if proportional control is used, the system becomes more responsive to the machining force. As the Bode diagrams are scaled to 0 dB in order to compare the bandwidth, this information goes lost. However the part displacement becomes smaller then zero, which means that lift-off occurs, which is unacceptable for good manufacturing practice. The results of this simulation should be with the specifications for the PI P-239.90 [12]. The actuation force F_{act} never exceeds the maximum specified force of 4500 N, but the maximum travel range (actuation stroke) of 0.18 mm is violated for all control strategies, except for open loop control and FFB control with a double LaF used a compensator. For this theoretical design it is therefore advisable to increase the motion amplification factor r_b / r_a and utilize a different PEA, like e.g. the PI P-247 [12].

4. CONCLUSIONS

Several feedback control techniques are compared in this study and their advantages and disadvantages are considered in the application to active clamps. The main findings of this study are that position feedback can be used to minimize unnecessary displacement of the workpiece, as it yields a smaller part displacement x_w when compared with force feedback (FFB) control. FFB is naturally stable, but it has a lower bandwidth than position feedback control. It should be noted that the application of only proportional control for position feedback control creates badly damped (“nervous”) systems, whereas application of a (double) lag filter (LaF) as compensator can be successfully used to stabilize the part-fixture system.

The dynamic simulation for the analysis of the control systems can also be used as a design verification tool, to check whether the system can perform well within its physical limitations, caused by e.g. limited stiffness, actuator bandwidth and maximum actuation stroke.

A design rule is established as well. Since piezoelectric materials are very stiff, compared to steel, they should preferably be placed directly on the part. This would mean that the PEA directly steers the part instead of the clamper-material between part and actuator.

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